Welcome to AP Calculus AB!

This assignment is designed to help you prepare for your upcoming AP Calculus AB class. All of the topics explored in this assignment should be a review of material you learned in your pre-calculus course; however, there may be a few topics that you may not have spent much time practicing in your prior math classes (for example, simplifying rational expressions).

I strongly encourage you try your best to work through this assignment: take note of the questions you may have had a difficult time completing and return to school ready to review! We will spend the first week of school reviewing this content, but we will not have enough time to do ALL of the questions so please come prepared with YOUR specific questions.

I'm very much looking forward to exploring AP Calculus AB with you this year! This course can be challenging, but I have full confidence in you: YOU CAN DO IT! ☺

If you have any questions or concerns, please feel free to contact at hagelka@lc-ps.org

Good luck! ☺
Ms. Hagel

Prerequisites for Calculus AB:

The following is a list of concepts that you have learned in one (or more) of your math classes throughout your school career. The summer assignment will not review ALL of these topics, but you will be expected to know ALL of them. Please be sure to review and practice these topics until you are confidently comfortable with all of them. You are permitted to use any resources you wish – previous class notes, online resources, textbooks, etc. – but please be aware that you are NOT to use a calculator to complete this summer assignment.

Prior Mathematical Knowledge:

- Slopes of horizontal and vertical lines
- How to calculate slope of lines
- How to write a linear equation in point-slope-form
- How to find x- and y-intercepts and points of intersection
- Vertical line test for a function
- Graphs of "parent" functions
- How to recognize horizontal and vertical shifts from an equation
- Doman and range
- How to graph piecewise functions
- How to write a piecewise function
- Function notation
- Even/odd functions and symmetry
- How to create and recognize composed functions
- Exponent rules
- Basic formula for exponential growth and decay: $y = Ce^{kt}$

AP Calculus AB Summer Assignment – Delta Math

- Graphs for exponential growth and decay: $y = e^x \ and \ y = e^{-x}$
- Use formula for compounded interest: $y = A\left(1 + \frac{i}{n}\right)^{nt}$
- Use formula for continuously compounded interest: $y = Pe^{rt}$
- Definition of "one-to-one" functions
- Relationship between one-to-one and inverse functions
- How to find inverse functions
- Relationship between domain and range of inverse functions
- Graphs for logarithms: $y = lnx$
- How to use natural logs to solve equations
- Values of six basic trig functions around the unit circle
- Graph of six basic trig functions
- Graphs (and domain and range) of inverse trig functions
- Solving inverse trigonometric equations
- Ratio, reciprocal, Pythagorean, and double angle identities (see below)

Trigonometric Identities: (MEMORIZE THESE!!! ☺)

Ratio Identities:

$\tan^a x = \frac{\sin^a x}{\cos^a x}$ and $\cot^a x = \frac{\cos^a x}{\sin^a x}$

Reciprocal Identities:

$$\frac{\sin^a x}{1} = \frac{1}{\csc^a x} \Leftrightarrow \frac{\csc^a x}{1} = \frac{1}{\sin^a x}$$
$$\frac{cos^a x}{1} = \frac{1}{\sec^a x} \Leftrightarrow \frac{\sec^a x}{1} = \frac{1}{\cos^a x}$$
$$\frac{\tan^a x}{1} = \frac{1}{\cot^a x} \Leftrightarrow \frac{\cot^a x}{1} = \frac{1}{\tan^a x}$$

Pythagorean Identities:

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Double-Angle Identities:

$$sin2x = 2sinxcosx$$
$$cos2x = cos^2x - sin^2x$$
$$cos2x = 1 - 2sin^2x$$
$$cos2x = 2cos^2x - 1$$

AP Calculus AB Summer Assignment

The summer homework assignment will be completed through Delta Math. In order to access the assignment you must create a Delta Math account at www.deltamath.com. (If you already have an account from a previous math course, you DO NOT need to create a new account.)

Once you have created an account, login to sign up for my class using the following information.
Teacher Code: 399193
Class Name: AP Calculus AB 20/21

You will know you have successfully signed up for my class when you can see the 6 Sections of the Summer Assignment. (See titles for assignment sections below.) The AP Calculus Summer Assignment will be active throughout the summer, but I encourage you to pace yourself while working through each section. This assignment meant as a review to help prepare you for the upcoming school year, so I suggest working on it closer to the beginning of next school year. I have provided a suggested pacing guide below, but you may work at whatever pace feels comfortable to you.

All sections of the AP Calculus Summer Assignment are DUE SEPTEMBER 4th, 2020 at 11:59 pm.

Section Number	Assignment Title:	Suggested Pacing:
1	AP Calculus Summer Assignment – Functions and Lines	7/27/20 to 7/31/20
2	AP Calculus Summer Assignment – Factoring and Polynomials	8/3/20 to 8/7/20
3	AP Calculus Summer Assignment – Radicals and Exponents	8/10/20 to 8/14/20
4	AP Calculus Summer Assignment – Rationals	8/17/20 to 8/21/20
5	AP Calculus Summer Assignment – Trigonometry*	8/24/20 to 8/28/20
6	AP Calculus Summer Assignment – Limits	8/31/20 to 9/4/20

All sections of the summer assignment should be completed WITHOUT the use of a calculator (*with the exception of the "Rotate Radians and Find Value" assignment under the Trigonometry section). The summer assignment can be easily completed using either a desktop/laptop computer, but it can also be completed using any hand-held device or tablet that has internet access.

If you **DO NOT** have access to technology at home during the summer, please complete the **"AP Calculus AB Alternative Summer Assignment"** provided below. (No calculators permitted on the alternative summer assignment.) Please note that this is an ALTERNATIVE assignment for those who DO NOT have technology at home during the summer – please complete the Delta Math Assignments if you are able to do so.

All students are welcome to complete the alternative summer assignment if they would like additional practice, but the alternative summer assignment will not be collected/graded if the Delta Math assignment is complete. If the Delta Math Assignment is NOT complete, the alternative assignment will be collected/graded on the first day of school.

AP Calculus AB ALTERNATIVE Summer Assignment

Finding Intercepts and Points of Intersection:

Find the intercepts. Remember, to find the *x*-intercept, substitute a "0" for *y* and solve for *x*.
To find the *y*-intercept, substitute a "0" for *x* and solve for *y*.

1. $y = 2x - 3$

2. $y = x^2 + x - 2$

3. $y = x^2\sqrt{9 - x^2}$

4. $y = \dfrac{x-1}{x-2}$

5. $x^2y - x^2 + 4y = 0$

Find the point(s) of intersection of the graphs. Remember that means you need to solve the system of equations, using either the linear combination or the substitution method.

6. $x + y = 2,\ 2x - y = 1$

7. $x + y = 7,\ 3x - 2y = 11$

8. $x^2 + y^2 = 5,\ x - y = 1$

9. $y = x^3,\ y = x$

10. $y = x^3 - 2x^2 + x - 1,\ y = -x^2 + 3x - 1$

Function Notation:

Remember that for function notation, the value in the parentheses replaces EACH "*x*" that appears in the function.

1. Given: $f(x) = 2x - 3$
   Find:
   a. $f(0)$ b. $f(-3)$
   c. $f(b)$ d. $f(2+h)$

2. Given: $f(x) = \sqrt{x+3}$
   Find:
   a. $f(-2)$ b. $f(6)$
   c. $f(c)$ d. $f(x+h)$

3. Given: $f(x) = 3x - 1$
   Find: $\dfrac{f(x) - f(1)}{x-1}$

4. Given: $f(x) = \dfrac{1}{x-3}$
   Find: $\dfrac{f(x) - f(5)}{x-5}$

5. Find the value of $\dfrac{f(1+h) - f(1)}{h}$ for each of the following functions:
   a. $f(x) = x$ b. $f(x) = x^2$
   c. $f(x) = x^3$ d. $f(x) = \dfrac{1}{x}$

6. Find the value of $\dfrac{f(x+h) - f(x)}{h}$ for each of the following functions:
   a. $f(x) = x^2 + x$ b. $f(x) = x - 2$
   c. $f(x) = x^2 - x + 1$ d. $f(x) = \dfrac{1}{x-2}$

Function Composition:

Find the composed function $h(x)$, where $h(x) = f(g(x))$. Remember that this means to take function $g(x)$ and put it INTO function $f(x)$ wherever an "x" appears.

1. $f(x) = \sin x$ and $g(x) = 3x + 1$

2. $f(x) = \sqrt{x}$ and $g(x) = \cos x$

3. $f(x) = \cos x$ and $g(x) = \sin x$

4. $f(x) = \sqrt{x}$ and $g(x) = 1 + x^2$

Name the two functions $f(x)$ and $g(x)$ that create the composed function $h(x) = f(g(x))$.

5. $h(x) = (2x - 5)^4$

6. $h(x) = \sec^5 x$

7. $h(x) = \csc x^4$

Find the composed function $h(x)$, where $h(x) = f(g(k(x)))$. This might be easiest in steps: first put $k(x)$ INTO $g(x)$, and then put your "new" $g(x)$ INTO $f(x)$.

8. $f(x) = \sqrt{x}$, $g(x) = \tan x$, and $k(x) = 5x$

9. $f(x) = \cot x$, $g(x) = 5 - \sin x$, and $k(x) = 2x$

Name the three functions $f(x)$, $g(x)$, and $k(x)$ that create the composed function $h(x) = f(g(k(x)))$.

10. $h(x) = \cos^2(3x - 2)$

Proving Identities Using Trigonometric Functions:

You also need to be able to recognize situations where substitutions with the basic identities (see list on first page of this packet) would be appropriate. The easiest way to review them is to practice proving identities. Remember that you can only make substitutions on ONE side of the equal sign, and then you try to continue to make substitutions and simplify until both sides are equal.

1. $\frac{\csc^2 x-1}{\cot^2 x} = 1$

2. $(1+\tan^2 x)(1-\sin^2 x) = 1$

3. $\sec^2 x = \frac{\sin^2 x+\cos^2 x}{\cos^2 x}$

4. $\frac{1}{\cot^2 x+1} = 1-\cos^2 x$

5. $\frac{\sec^2 x}{\sec^2 x-1} = \csc^2 x$

6. $\csc^2 x\tan^2 x - 1 = \tan^2 x$

7. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

8. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

9. $\tan x = \frac{\sin 2x}{1+\cos 2x}$

10. $\cot x = \frac{\sin 2x}{1-\cos 2x}$

11. $\frac{\sin 2x}{2\sin x} = \cos x$

12. $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

Graphs of Parent Functions:

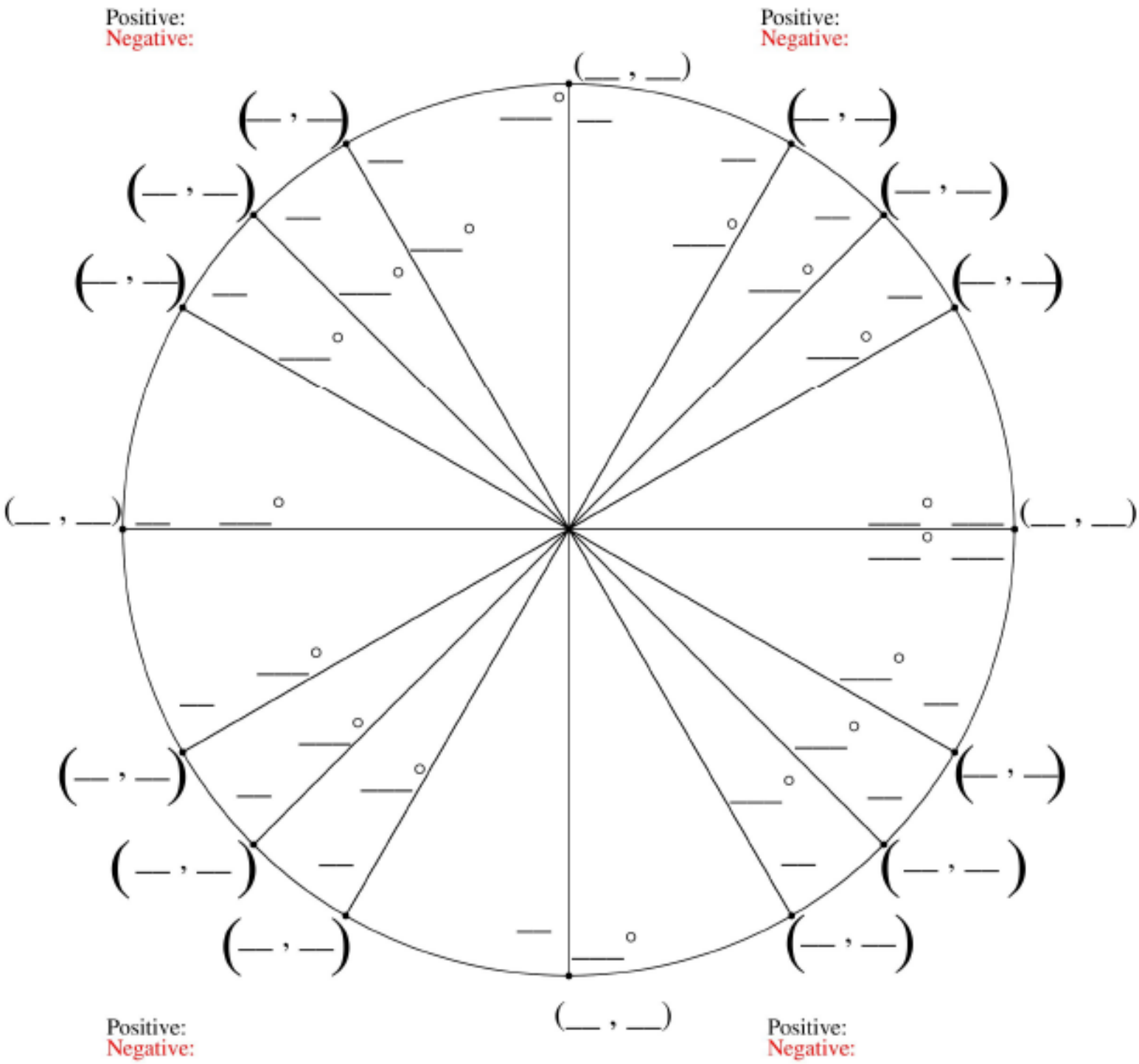
The "parent functions" are basic functions that you should be able to (a) RECOGNIZE in equation form, and (b) VISUALIZE as a graph. You also should be aware of how addition, subtraction, multiplication, or division can affect the equation and result in translations or shifts in the graph. Make a simple sketch of each "parent" function. If you don't know what the graph looks like, use your calculator for now, but LEARN what these functions look like!

$y = x^2$	$y = x^3$	$y = \lvert x \rvert$
$y = \sqrt{x}$	$y = \sqrt[3]{x}$	$y = x^{2/3}$
$y = e^x$	$y = e^{-x}$	$y = \frac{1}{x}$
$y = \ln x$	$y = \sqrt{1 - x^2}$	$y = \frac{1}{x^2}$

$y = \sin x$	$y = \cos x$	$y = \tan x$
$y = \csc x$	$y = \sec x$	$y = \cot x$
$y = \arcsin x$ (also known as $y = \sin^{-1} x$)	$y = \arccos x$ (also known as $y = \cos^{-1} x$)	$y = \arctan x$ (also known as $y = \tan^{-1} x$)
$y = \operatorname{arc}\csc x$ (also known as $y = \csc^{-1} x$)	$y = \operatorname{arc}\sec x$ (also known as $y = \sec^{-1} x$)	$y = \operatorname{arc}\cot x$ (also known as $y = \cot^{-1} x$)

The Unit Circle:

Use this unit circle to help you visualize the angles and recall the values of the six trigonometric functions for standard angles measures on the unit circle. Remember the x-value identifies the exact cosine value and the y-value identifies the exact sine value of the given angle.



Exact Values on the Unit Circle:

Provide the radian equivalent for each standard angle measure on the unit circle, and then complete the table of values for all six trigonometric functions. Redo this worksheet several times (just use scrap paper)…hopefully you'll get better and faster each time! ☺

Degrees	Radians	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
0°							
30°							
45°							
60°							
90°							
120°							
135°							
150°							
180°							
210°							
225°							
240°							
270°							
300°							
315°							
330°							
360°							

Adding & Subtracting Rational Expressions:

A complex fraction is one that contains fractions within the fraction itself. Simplify the complex fractions below. Remember that you divide fractions by *leaving the numerator the same* and *multiplying by the reciprocal of the denominator*.

1. $\dfrac{\frac{x-9}{8}}{\frac{3-x}{12}}$

2. $\dfrac{\frac{x+y}{5}}{\frac{x^2+y^2}{5}}$

NOTE: What happens when the denominators are the same?

3. $\dfrac{\frac{x+y}{2x-y}}{\frac{x+y}{2x+y}}$

NOTE: What happens when the numerators are the same?

Simplify the complex fractions below. Remember that because of the addition and subtraction, you need to *find a common denominator on the top first*, and *then find a common denominator on the bottom*. Only THEN can you perform the division!

4. $\dfrac{\frac{1}{x}-\frac{1}{y}}{1+\frac{1}{x}}$

5. $\dfrac{x+\frac{x}{3}}{x-\frac{x}{6}}$

6. $\dfrac{\frac{1}{x}+2}{\frac{3}{x}-4}$

7. $\dfrac{\frac{2x}{2x+1}-1}{1+\frac{2x}{1-2x}}$

8. $\dfrac{3+\frac{5}{x+2}}{3-\frac{10}{x+7}}$

9. $\dfrac{(x+y)\left(\frac{1}{x}-\frac{1}{y}\right)}{(x-y)\left(\frac{1}{x}+\frac{1}{y}\right)}$

Remember that negative exponents become positive when they are "shifted" to the opposite "side" of a fraction. "Clean up" negative exponents in each term first, and then find the common denominator.(NOTE: These problems have all been taken directly from actual calculus problems!)

10. $\frac{1}{3}x^{-2/3}-\frac{7}{3}x^{7/3}$

11. $\frac{9}{4}x^{5/4}+\frac{1}{4}x^{-3/4}$

12. $\frac{3}{2}x^{1/2}+2x^{-3/2}$

These can get extremely messy! My recommendation is to (a) "clean up" any fractions and negative exponents on the left side of the numerator, (b) "clean up" any fractions and negative exponents on the right side of the numerator, (c) find the common denominator of the numerator, and then (d) combine the complex fraction like Exercises #1-3 above. (NOTE: These problems also have all been taken directly from parts of actual calculus problems!)

13. $\dfrac{(3x+1)\left(\frac{1}{2}\right)\left(x^{-1/2}\right)-\left(x^{1/2}\right)(3)}{(3x+1)^2}$

14. $\dfrac{(x^3+1)\left(\frac{1}{3}\right)\left(x^{-2/3}\right)-\left(x^{1/3}-1\right)(3x^2)}{(x^3+1)^2}$

15. $\dfrac{(x^4+1)\left(\frac{1}{4}\right)\left(x^{-3/4}\right)-\left(x^{1/4}-1\right)(4x^3)}{(x^4+1)^2}$